

On electron channeling and the de Broglie internal clock

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Abstract

Electron channeling in silicon crystals has brought forward the possibility of having detected the particle's "de Broglie internal clock", as giving rise to the observed resonance peak at the center of the expected transmission probability dip. A classical multiple scattering calculation fails to represent the experimental results unless, surprisingly, the interaction frequency is twice the de Broglie's clock frequency, that is, the "Zitterbewegung" frequency. In the present paper, the observed characteristics of this process are shown to be consistent with a free particle quantum mechanical motion described by Dirac's Hamiltonian.

I. INTRODUCTION

Electron channeling in silicon crystals[1, 2] has brought forward the possibility of having detected a particle's "internal clock", as an intrinsic oscillation whose frequency is given by de Broglie's daring association $h\nu = m_0c^2$, where h is Planck's constant, m_0 is the particle rest mass and c the speed of light in vacuum.[3] More recently, a clock linked to this relation - referred as "de Broglie internal clock", "Compton clock" or "de Broglie periodic phenomenon" - has been demonstrated using an optical frequency to self-reference a Ramsey-Bordé atom interferometer.[4]

The cited channeling experiments in which relativistic electrons are aligned along a major axial direction of a thin single crystal, do exhibit a reduced transmission probability with respect to neighboring angles. At certain energies however, a central sharp peak appears in the atomic row direction. The pattern observed is a "W" instead of a "U". This central peak is attributed to a resonance process denoted as "rosette motion", which results in a reduction of the multiple scattering effects for electrons moving parallel to a string of atoms with a momentum such that they pass atoms with a frequency equal to the de Broglie frequency. The expected consequence is a higher transmittivity relative to closely nearby directions and momenta, as a fraction of the electrons are trapped in a spiral motion about the atomic row which, projected onto the transverse plane, executes rosettes, i.e., bound orbits that precess.

A phenomenological calculation by a Montecarlo method is carried out[1, 2], in which the electron motion is described classically, based on a theoretical model applied successfully to the channeling process at relativistic energies[9, 10]. The basic assumption made is that the distance L travelled during a de Broglie laboratory period is equal to the interatomic distance d in the crystal row. This defines the group velocity v_{gp} and consequently the energy. Although giving an electron energy close to the experimental one, the theoretical calculation does not yield the experimental results. It is then noted that, surprisingly, the results are reproduced at the same energy if L is taken to be equal to twice the distance d . It is noted that this requires to consider a reduced period and consequently a higher frequency, namely the "Zitterbewegung" frequency.

In the present paper it is shown that all the observed characteristics of this process follow naturally from the quantum mechanical free-particle motion described by a Dirac

Hamiltonian (albeit with an effective mass resulting from the average interaction with the crystal atoms). A possible explanation of how the surprising fit arises is presented here.

II. THE FREE PARTICLE DIRAC HAMILTONIAN AS A SYMMETRY OPERATION

Consider the free particle Dirac Hamiltonian

$$H = c\boldsymbol{\alpha} \cdot \mathbf{p} + \beta m_0 c^2 \quad (1)$$

where $\alpha = (\alpha_x, \alpha_y, \alpha_z)$ and β are the Dirac matrices. Recalling that the infinitesimal ($\epsilon \ll 1$) unitary operator $S(\epsilon) = e^{\{i\epsilon \mathbf{p}/\hbar\}}$ acting on a position eigenstate yields a displaced eigenstate, namely[6]:

$$S(\epsilon)|x\rangle = e^{\{i\epsilon \mathbf{p}/\hbar\}}|x\rangle = [1 + (i\epsilon \mathbf{p}/\hbar) + \frac{1}{2}(i\epsilon \mathbf{p}/\hbar)^2 + \dots]|x\rangle = |x + \epsilon\rangle,$$

it follows that the infinitesimal unitary operator ($\tau \ll 1$):

$$\begin{aligned} U(\tau) &= e^{-i\tau H/\hbar} = e^{-i\tau\{c\boldsymbol{\alpha} \cdot \mathbf{p} + \beta m_0 c^2\}/\hbar} = \\ &= [1 + (i\tau c\boldsymbol{\alpha} \cdot \mathbf{p}/\hbar) + \frac{1}{2}(i\tau c\boldsymbol{\alpha} \cdot \mathbf{p}/\hbar)^2 + \dots] \exp^{-i\beta \tau m_0 c^2/\hbar} \end{aligned} \quad (2)$$

induces, in configuration space, a position displacement by an amount $\delta \mathbf{r} = \tau c\boldsymbol{\alpha}$ and a phase shift $\delta \phi = \beta(\tau m_0 c^2/\hbar)$, i.e.:

$$\Phi(\mathbf{r}) = \langle \mathbf{r} | \Phi \rangle = e^{i\phi} \varphi(\mathbf{r}) \rightarrow e^{i(\phi+\delta\phi)} \varphi(\mathbf{r} + \tau c\boldsymbol{\alpha}) \quad (3)$$

As $[U(\tau), H] = 0$, the displaced wave function satisfies the same Schrödinger equation. $U(\tau)$ is thus a symmetry operation. Finite displacements are achieved by repeated applications. The phase shift is seen to be related to the reduced de Broglie (or Compton) frequency ($m_0 c^2/\hbar$).

The Dirac relativistic free particle motion in the Heisenberg picture is given by:

$$\mathbf{r}(t) = \mathbf{r}(0) + (c^2 \mathbf{p}/H)t + (\hbar c/2iH)(e^{2iHt/\hbar} - 1)\mathbf{F} \quad (4)$$

with $\mathbf{F} = c\boldsymbol{\alpha} - (c^2 \mathbf{p}/H)$. Averaging over a general positive energy wave packet one has $\langle c^2 \mathbf{p}/H \rangle = \mathbf{v}_{gp}$, the group velocity, and:

$$\langle \mathbf{r}(t) \rangle = \langle \mathbf{r}(0) \rangle + \mathbf{v}_{gp}t + \langle (\hbar c/2iH)(e^{2iHt/\hbar} - 1)\mathbf{F} \rangle \quad (5)$$

Thus, as pointed out by Schrödinger, $\langle \mathbf{r}(t) \rangle$ is an oscillatory motion ("Zitterbewegung") about the rectilinear uniform motion $\langle \mathbf{r}(0) \rangle + \mathbf{v}_{gp}t$. The oscillation arises when both positive and negative energies are present in the wave packet, bringing in a contribution of the operator \mathbf{F} [7, 8]. The position coincides with the uniform motion at times t_n such that $2Ht_n/\hbar = 2\pi n$ with $n = 1, 2, \dots$, or equivalently at intervals $\Delta t = t_{n+1} - t_n = 2\pi\hbar/2H = h/2H = h/2m_0c^2\gamma$, where $\gamma = [1 - (\mathbf{v}_{gp}/c)^2]^{-1/2}$ is the Lorentz factor. This interval corresponds to the Zitterbewegung laboratory period, (denoted as Z period) $T_Z^{lab} = (1/\nu_Z^{lab})$, thus equal to half the laboratory de Broglie period (denoted as B period).

The phase at time t is:

$$\langle \phi(t) \rangle = \langle \beta(t) \rangle t (m_0c^2/\hbar) \quad (6)$$

where $\langle \beta(t) \rangle = \langle m_0c^2/H + e^{2iHt/\hbar}G \rangle$ with $G = \beta(0) - (m_0c^2/H)$, an operator that also connects only positive and negative energy eigenstates. Then at the coincidence times $t_n = 2\pi n\hbar/2H$, one has $\langle \beta(t_n) \rangle = \langle m_0c^2/H + G \rangle = \langle \beta(0) \rangle$ and the phase is:

$$\langle \phi(t_n) \rangle = \langle \beta(t_n) \rangle t_n (m_0c^2/\hbar) = \langle \beta(0) \rangle (2\pi n\hbar/2H)(m_0c^2/\hbar) = \langle \beta(0) \rangle (n\pi/\gamma) \quad (7)$$

Consequently the change in phase between two consecutive coincidence times is given by:

$$\Delta\varphi = \langle \beta(0) \rangle (\pi/\gamma) \approx 0 \quad (8)$$

when v_{gp} approaches c . Thus, at each coincidence time, a relativistic electron arrives with the same amplitude and phase.

III. ELECTRON CHANNELING

In a channeling set-up the rectilinear motion would be aligned along the crystal atomic row. Then the electron may be made to coincide with crystal atoms given the appropriate group velocity and consequently the appropriate energy. In the experiment cited[1, 2], the requirement was made that the electron should advance the interatomic distance d in one B period, i.e., $L = v_{gp}T_B^{lab} = d$. Now $T_B^{lab} = (h\gamma/m_0c^2)$ since the de Broglie linear frequency in the laboratory is $\nu_B^{lab} = (m_0c^2/h\gamma)$. [3, 5]. Then:

$$v_{gp} = (d/T_B^{lab}) = (d)(m_0c^2/h\gamma) \quad (9)$$

Defining $d(m_0c^2/hc) = \alpha$ and $(v_{gp}/c) = \beta$, it follows that:

$$(\beta\gamma)^2 = \beta^2(1 - \beta)^{-1} = \alpha^2,$$

and finally

$$\beta = (v_{gp}/c) = \alpha/(\alpha + 1)^{1/2} \approx 1 - 1/(2\alpha^2) \text{ if } \alpha \gg 1 \quad (10)$$

In the reported experiment $d = 3.84 \text{ \AA} = 3.84 (10^5) F$. Then

$$\alpha = d(m_0c^2)/hc = (1/2\pi)d(m_0c^2)/\hbar c = 158.2707 \quad (11)$$

and the motion is seen to be highly relativistic. The laboratory energy is:

$$E = m_0c^2\gamma \approx m_0c^2\alpha = m_0c^2d(m_0c^2/hc) = 0.511(158.2707) \text{ MeV} = 80.876 \text{ MeV} \quad (12)$$

The corresponding phase shift is from Eq.(8) close to zero, as $T_B^{lab} = 2T_Z^{lab}$. The electron located at one atom at a certain moment will reach the next one after one de Broglie period (or two Zitterbewegung periods) with the same amplitude and phase. This can be the base for a resonance phenomenon.

Note that the same energy value of 80.876 MeV results from requiring the interatomic distance d to be attained after two Z periods. On the other hand, from the above derivation, if the full interatomic distance d is to be attained after one Z period, or equivalently $2d$ in a B period, the energy must be $E = 161.752 \text{ MeV}$. Finally, a resonance energy $E = 40.438 \text{ MeV}$ follows from requiring the distance $d/2$ to be travelled in a B period. All these situations are recognized in Refs.(1, 2), suggesting that the phenomenon should also be observable at these energies.

IV. THE SEMI-CLASSICAL CALCULATION

As seen in Fig. 4 of Ref.(1) the classical phenomenological calculation does not reproduce the experimental result. On the other hand, taking $L = 2d$, surprisingly seems to succeed at the same energy, whereas one would expect a group velocity twice as large and an energy of 161.752 MeV, as noted above and also in Ref.1.

A possible explanation may be the following. The calculation has as independent parametrs the intensity K of confining potential $U(x_i, y_i, z_i) = K x_i^2 y_i^2 z_i^2$, and the range

σ of the Gaussian shape used to introduce a fluctuating length at each step. The deviation angle at each step in the x -direction of the transverse plane for $L = d$ is given as :

$$d\alpha_i = 2LK y_i^2 z_i^2 x_i / m = 2dK y_i^2 z_i^2 x_i / m \quad (13)$$

and similarly in the y -direction (see Eqs.(2) and (3) of Ref. (1)). If one considers now $L = 2d$ one can rewrite this equation as

$$d\alpha_i = 2dK' y_i^2 z_i^2 x_i / m \quad (14)$$

with $K' = 2K$. This is equivalent to conserve $L = d$, and consequently the group velocity and energy, but to double the strength of the potential and thus the restoring force. This results in a contraction of all dimensions. As such, it can be expected to reduce the width of the transmission dip.

In addition the normalized Gaussian was given the value $\sigma = L/4 = d/4$. If now $L = 2d$, the range parameter becomes $\sigma' = d/2$, increasing its range and reducing the maximum value by $1/2$. As pointed out in Ref.(1), "the parameter σ acts only on the dip amplitude, not on its width". Using a more extended Gaussian function will reduce the amplitude of the transmission dip. Thus the above changes in K and σ are expected to reduce both the width and the amplitude of the transmission dip, bringing the calculation in closer agreement with the experiment, as seems to have occurred in Ref.1.

Finally, the remainig difference between the theoretical and the experimental resonant energies could be accounted by considering that the electron's motion in the crystal is well represented by the free-particle Dirac Hamiltonian, albeit with an effective mass m^* instead of m_0 that takes into account the average interaction of the electron with the crystal atoms[13, 14]. Following this assumption through the above derivation yields $(m^*/m_0)^2 = (81.1)/(80.876) = 1.00225$ and $m^* = 1.0013 m_0$.

V. CONCLUSION

The resonant conditions observed in the significative electron channeling experiment considered have been shown to be consistent with Dirac's quantum mechanical description of the free-particle motion. An explanation is offered on how the modified calculation presented in the experimental paper[1, 2] came to be more in agreement with the observed

result. Notwithstanding its acceptance, it is to be stressed that the experiment provides an indirect evidence of the Zitterbewegung phenomenon, and of the de Broglie clock, in the case of electrons, where direct observation is still beyond present technical capabilities. At present Zitterbewegung has indeed been observed in experimental conditions whose dynamics simulate Dirac's Hamiltonian[11, 12].

VI. BIBLIOGRAPHY

- [1] Catillon, P. et al., A Search for the de Broglie Particle Internal Clock by means of Electron Channeling, *Found.Phys.* **38**, 659-664 (2008)
- [2] Gouanère, M. et al., Experimental observation compatible with the particle internal clock, *Annales de la Fondation Louis de Broglie* **30**, 109-114 (2005)
- [3] de Broglie, L., *Recherches sur la théorie des quanta*, Réédition du texte de 1924, Masson & Cie, Paris (1963)
- [4] Lan, S-Y. et al., A Clock Directly Linking Time to a Particle Mass, *Scienceexpress* 10.1126/science.1230767 (2013)
- [5] Lochak, G., Addendum au précédent mémoire sur la fréquence propre de de Broglie, *Annales de la Fondation Louis de Broglie* **30**, 115-117 (2005)
- [6] Messiah, A., *Quantum Mechanics*, John Wiley & Sons, Inc. (New York-London-Sidney) (1966)
- [7] Greiner, W., *Relativistic Quantum Mechanics* (3d ed.), Springer-Verlag Berlin Heidelberg New York (2000)
- [8] Thaller, B., *The Dirac Equation*, Springer-Verlag Berlin Heidelberg (1992)
- [9] Lindhard, J., Influence of crystal lattice on motion of energetic charged particles, *Mat.Fys.Medd.Dan Vid.Selsk.* **34**, 1 (1965)
- [10] Gemmel, D.S., Channeling and related effects in the motion of charged particles through crystals, *Rev.Mod.Phys.* **46**, 129-227 (1974)
- [11] Gerritsma, R., *et al.*, Quantum simulation of the Dirac equation, *Nature* **463**, 68-71 (2010)
- [12] LeBlanc, L.J. *et al.*, Direct observation of zitterbewegung in a Bose-Einstein condensate, *New Journal of Physics* **15**, 073011 (2013)

- [13] Kittel, C., Introduction to Solid State Physics (3d ed.), John Wiley&Sons, Inc., New York, London, Sydney (1968)
- [14] Green, M.A., Intrinsic concentration, effective densities of states, and effective mass in silicon, J.Appl.Phys. **67**, 2944 (1990)